Towards a solution of the charmonium production controversy: $k_\perp$-factorization versus color octet mechanism

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The cross section of χ_{cJ} hadroproduction is calculated in the $k_\perp$ -factorization approach. We find a significant contribution of the χ_{c1} state due to non-applicability of the Landau-Yang theorem because of off-shell gluons. The results are in agreement with data and leave no room for the color-octet contribution. Our results could therefore lead to a solution of the longstanding controversy between the color singlet model and the color octet mechanism.

The production of heavy quarkonia received a lot of attention from both theory and experiment in recent years. It is e.g. the most prominent signal in the search for the quark gluon plasma. Its usefullness is, however, questionable as long as the charmonium production process is not understood. For a review we refer to [1–3]. Originally heavy quarkonium production was described in the color singlet model (CSM) [4,5]. Calculations based on this model and standard collinear factorization show however disagreement with the experimental data. For example the next-to-leading order (NLO) QCD collinear results for direct J/Ψ hadroproduction underestimate the measured cross section at Tevatron by a factor of ≈ 50 (see fig.4 in [6] and Ref. [7]). The proposed solution to this strong discrepancy is the so called color-octet-mechanism (COM) [8,9], according to which a color octet $q\overline{q}$-pair which has been produced at short distances can evolve into a physical quarkonium state by radiating soft gluons. The COM introduces uncalculable non-perturbative parameters, the color octet matrix elements, which have to be determined by a fit to the data [10,11]. The inclusion of the COM into NLO QCD collinear calculations leads in the case of hadroproduction to a reasonable agreement with experiment [10,11]. In these calculations the color octet contribution dominates.

On the other hand up to now the COM suffers at least from two unsolved problems. When the, supposedly universal, color octet matrix elements are applied to electroproduction of heavy quarkonium the theoretical predictions fail to describe the data [12]. Furthermore the results of the COM for polarized heavy quarkonium hadroproduction seem to be incompatible with recent data from Tevatron [13].

The longstanding discrepancy between the results based on the CSM together with collinear factorization and the experimental data shows up especially strongly in the $k_\perp$-dependent cross sections from Tevatron [6]. Thus one can wonder if the collinear approximation, in which in NLO the only transverse momentum of the produced quarkonium comes from an additional final state gluon, is suitable at all.

The aim of our paper is to clarify this question by a study of χ_{cJ} hadroproduction within the $k_\perp$ -factorization Regge approach, which takes the nonvanishing transverse momenta of the colliding t-channel gluons into account. More precisely we calculate the production of J/Ψ's originating from radiative χ_{cJ} decays. In a recent study [14] of open $b\bar{b}$ hadroproduction we found that $k_\perp$ -factorization gives far better results than NLO collinear QCD calculations and we expect a similar improvement for heavy quarkonium production. The main ingredients of our calculations in [14] are the unintegrated gluon distribution and the effective next-to-leading-logarithmic-approximation (NLLA) $q\overline{q}$ -BFKL production vertex which we use in this article as well. The projection of the heavy quark-antiquark pair onto the corresponding charmonium state is described in the standard way within the non-relativistic-quarkonium-model [5,4,10,11].

We study the production of χ_{cJ} whose lowest Fock state component is $q\overline{q}(^3P_J)$. For J/Ψ (a $q\overline{q}(^3S_1)$ state) the LO production amplitude is zero. In order to get a nonzero $q\overline{q}(^3S_1)$-amplitude one has (in NLO in α_S) to emit an additional gluon. The amplitude for the production of a $q\overline{q}$ -pair plus a gluon within the BFKL approach would in our case require an effective three-particle production vertex which still has to be derived. In contrast the production of a χ_{c1} can be calculated in our approach in LO because the Landau-Yang theorem which usually forbids the production of a $^3P_1$ state is not valid for off-mass-shell gluons.

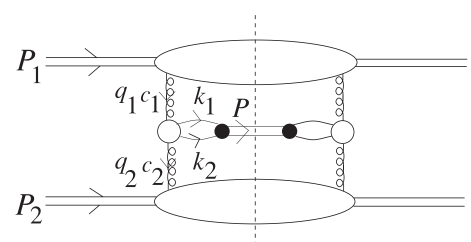

FIG. 1. The basic diagram

We use the following definition of the light cone coordinates

$$k^+ = k^0+k^3,\ k^- = k^0-k^3,\ k_\perp = (0,k^1,k^2,0) = (0,\mathbf{k},0).$$

In the c.m. frame the momenta of the scattering hadrons are given by

$$P_1^+ = P_2^- = \sqrt{s},\quad P_1^- = P_2^+ = P_{1\perp} = P_{2\perp} = 0,$$

where the Mandelstam variable s is as usual the c.m.s. energy squared. The momenta of the t-channel gluons are q_1 and q_2 (see Fig.1). The on-shell quark and antiquark (with mass m) have momentum k_1 respectively k_2 with

$$k_1^- = \frac{(m^2 - k_{1\perp}^2)}{k_1^+},\quad k_2^- = \frac{(m^2 - k_{2\perp}^2)}{k_2^+}.$$

In the high energy (large s) regime we have

$$P^+ = q_1^+ - q_2^+ \approx q_1^+,\ P^- = q_1^- - q_2^- \approx -q_2^-,$$
$$q_1^2 \approx q_{1\perp}^2,\ q_2^2 \approx q_{2\perp}^2,$$

where $P = k_1+k_2$ is the momentum of the heavy quarkonium with $P^2 = 4m^2$. The longitudinal momentum fractions of the gluons are $x_1 = q_1^+/P_1^+$, $x_2 = -q_2^-/P_2^-$.

The heavy quarkonium hadroproduction cross section in the $k_\perp$-factorization approach is [17], [18]

$$\sigma_{P_1P_2\to\chi X} = \frac{1}{8(2\pi)}\int\frac{d^3P}{P^+}d^2q_{1\perp}d^2q_{2\perp}\delta^2(q_{1\perp}-q_{2\perp}-P_\perp)$$
$$\mathcal{F}(x_1,q_{1\perp})\frac{1}{(q_{1\perp}^2)^2}\left\{\frac{\psi_\chi^{\dagger c_2c_1}\psi_\chi^{c_2c_1}}{(N_C^2-1)^2}\right\}\frac{1}{(q_{2\perp}^2)^2}\mathcal{F}(x_2,q_{2\perp}). \quad (1)$$

The factor $(N_C^2-1)^2$ comes from the projection on color singlet in the t-channel. $\mathcal{F}(x,q_\perp)$ is the unintegrated gluon distribution. The heavy quarkonium production amplitude $\psi_\chi^{c_2c_1}(x_1,x_2,q_{1\perp},q_{2\perp},P)$ is factorized (see below) in a hard part which describes the production of the $q\overline{q}$ pair and an amplitude describing the binding of this pair into a physical charmonium state. We choose the scale μ^2 for $\alpha_S(\mu^2)$ in the amplitude $\psi_\chi^{c_2c_1}$ to be $\mathbf{q}_1^2 = -q_{1\perp}^2$ respectively $\mathbf{q}_2^2 = -q_{2\perp}^2$ [19].

The amplitude for the production of the charmonium state can be written as

$$\psi_\chi^{c_2c_1} = \mathcal{P}(q\overline{q}\to\chi_{cJ})\bullet\Psi^{c_2c_1}. \quad (2)$$

The $q\bar{q}$ production vertex $\Psi^{c_2c_1}$ derived in [20] for massless QCD, appropriately generalized for massive quarks, has the form

$$\Psi^{c_2c_1} = -g^2\left(t^{c_1}t^{c_2}\,b(k_1,k_2) - t^{c_2}t^{c_1}\,b^T(k_2,k_1)\right),$$

where t^c are the colour group generators in the fundamental representation. The operator $\mathcal{P}(q\overline{q}\to\chi_{cJ})$ projects the $q\bar{q}$ pair onto the charmonium bound state, see below.

The functions $b(k_1,k_2)$ and $b^T(k_2,k_1)$ are illustrated in Fig.2 and their explicit form can be found in [14]. One important property of the charmonium production amplitude for on-mass-shell quark and antiquark states (2), which is related to the gauge invariance of the whole approach, is its vanishing in the limit $q_{1\perp}\to 0$ (or $q_{2\perp}\to 0$).

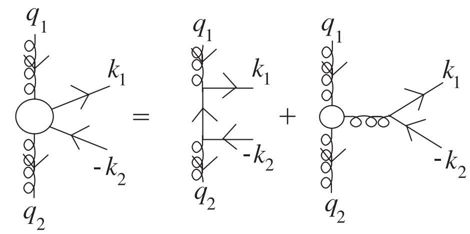

FIG. 2. The effective vertex

The relation between the usual gluon distribution $xg(x,\mathbf{q}^2)$ and the unintegrated gluon distribution $\mathcal{F}(x,\mathbf{k})$ is given by

$$xg(x,\mathbf{q}^2) = \int_0^\infty\frac{d\mathbf{k}^2}{\mathbf{k}^2}\Theta(\mathbf{q}^2-\mathbf{k}^2)\mathcal{F}(x,\mathbf{k}). \quad (3)$$

$\mathcal{F}(x,\mathbf{k})$ includes the evolution in x and $\mathbf{k}^2$ described by the BFKL and DGLAP equation. In the non-perturbative region of small $\mathbf{k}^2$ the unintegrated gluon distribution is not known, therefore we write (3) according to [21,18,22,23] as

$$xg(x,\mathbf{q}^2) = xg(x,\mathbf{q}_0^2) + \int_{\mathbf{q}_0^2}^\infty\frac{d\mathbf{k}^2}{\mathbf{k}^2}\Theta(\mathbf{q}^2-\mathbf{k}^2)\mathcal{F}(x,\mathbf{k}),$$

which introduces the a priori unknown initial scale $\mathbf{q}_0$ and the initial gluon distribution $xg(x,\mathbf{q}_0^2)$. Following [21,22], we neglect the momentum dependence of the hard cross section in the soft region $|\mathbf{q}|<|\mathbf{q}_0|$, so that

$$\frac{1}{q_{1\perp}^2}\left\{\frac{\psi_\chi^{\dagger c_2c_1}\psi_\chi^{c_2c_1}}{(N_C^2-1)^2}\right\}\frac{1}{q_{2\perp}^2} \equiv S(q_{1\perp},q_{2\perp}) \to$$
$$\left[S(q_{1\perp},q_{2\perp})\Theta(\mathbf{q}_2^2-\mathbf{q}_0^2) + S(q_{1\perp},0)\Theta(\mathbf{q}_0^2-\mathbf{q}_2^2)\right]\Theta(\mathbf{q}_1^2-\mathbf{q}_0^2)$$
$$+\left[S(0,q_{2\perp})\Theta(\mathbf{q}_2^2-\mathbf{q}_0^2) + S(0,0)\Theta(\mathbf{q}_0^2-\mathbf{q}_2^2)\right]\Theta(\mathbf{q}_0^2-\mathbf{q}_1^2),$$

see also the discussion of this expression in [14].

One important point is the proper choice of the unintegrated gluon distribution function. We use the results of Kwiecinski, Martin and Staŝto [15]. They determined it using a combination of DGLAP and BFKL evolution equations. With the initial conditions

$$\mathbf{q}_0^2 = 1\text{ GeV},\ \ xg(x,\mathbf{q}_0^2) = 1.57(1-x)^{2.5}. \quad (4)$$

they obtained an execellent fit to $F_2(x,Q^2)$ data over a large range of x and Q^2.

The easiest way to perform the calculation of the amplitude in the color singlet case is to adapt the method of [4,5]. The projection of the hard amplitude onto the charmonium bound state is given by

$$\psi_{\chi}^{c_2c_1} = \mathcal{P}(q\bar{q} \to \chi_{cJ}) \bullet \Psi^{c_2c_1}$$
$$= \sum_{i,j} \sum_{L_z,S_z} \frac{1}{\sqrt{m}} \int \frac{d^4q}{(2\pi)^4} \delta\left(q^0 - \frac{\vec{q}^2}{M^2}\right) \Phi_{L=1,L_z}(\vec{q})$$
$$\langle L=1, L_z, S=1, S_z | J, J_z \rangle \langle 3i, \bar{3}j | 1 \rangle \, Tr\left\{ \Psi_{ij}^{c_2c_1} \mathcal{P}_{S=1,S_z} \right\}, \tag{5}$$

where $\Phi_{L=1,L_z}(\vec{q} = \vec{k}_1 - \vec{k}_2)$ is the momentum space wave function of the charmonium, and the projection operator $\mathcal{P}_{S=1,S_z}$ for a small relative momentum $q = k_1 - k_2$ has the form

$$\mathcal{P}_{S=1,S_z} = \frac{1}{2m}(\not{k}_2 - m)\frac{\not{\epsilon}(S_z)}{\sqrt{2}}(\not{k}_1 + m).$$

The Clebsch-Gordan coefficient in color space is given by $\langle 3i, \bar{3}j | 1 \rangle = \delta_{ji}/\sqrt{N_C}$. Since P-waves vanish at the origin, one has to expand the trace in (5) in a Taylor series around $\vec{q} = 0$. This yields an expression proportional to

$$\int \frac{d^3\vec{q}}{(2\pi)^3} q^\alpha \Phi_{L=1,L_z}(\vec{q}) = -i\sqrt{\frac{3}{4\pi}} \epsilon^\alpha(L_z) \mathcal{R}'(0),$$

with the derivative of the P-wave radial wave function at the origin $\mathcal{R}'(0)$ whose numerical values can be found in [16]. For the individual $\chi_{cJ=1}$ and $\chi_{cJ=2}$ amplitudes we use

$$\sum_{L_z,S_z} \langle 1, L_z, 1, S_z | 1, J_z \rangle \, \epsilon^\mu(L_z) \epsilon^\nu(S_z) = -i\sqrt{\frac{1}{2}} \varepsilon^{\mu\nu\alpha\beta} \frac{P_\alpha}{M} \epsilon_\beta(J_z)$$
$$\sum_{L_z,S_z} \langle 1, L_z, 1, S_z | 2, J_z \rangle \, \epsilon^\mu(L_z) \epsilon^\nu(S_z) = \epsilon^{\mu\nu}(J_z)$$

where we introduce the spin 1 and spin 2 polarization tensors $\epsilon_\beta(J_z)$ and $\epsilon^{\mu\nu}(J_z)$ of the produced charmonium $\chi_{cJ=1}$ respectively $\chi_{cJ=2}$. In the unpolarized case the squared amplitudes are further evaluated using

$$\sum_{J_z} \epsilon^\mu(J_z) \epsilon^\nu(J_z) = -g^{\mu\nu} + \frac{P^\mu P^\nu}{M^2} = P^{\mu\nu},$$
$$\sum_{J_z} \epsilon^{\mu\nu}(J_z) \epsilon^{\alpha\beta}(J_z) = \frac{1}{2}\left(P^{\mu\alpha}P^{\nu\beta} + P^{\nu\alpha}P^{\mu\beta}\right) - \frac{1}{3} P^{\mu\nu} P^{\alpha\beta}.$$

The cross section for J/Ψ production from radiative χ_{cJ} decays is then given by [10,11]

$$\sigma_{J/\Psi \ from \ \chi_c} = \sum_{J=0,1,2} \sigma_{P_1P_2 \to \chi_{cJ} X} \cdot Br(\chi_{cJ} \to J/\Psi + \gamma),$$

with the χ_{cJ} hadroproduction cross section $\sigma_{P_1P_2 \to \chi_{cJ} X}$ (1). Because of the small branching ratio $Br(\chi_{cJ=0} \to J/\Psi + \gamma) = \mathcal{O}(10^{-3})$ the contribution from $\chi_{cJ=0}$ is negligible. For the numerical computation we use the values

$$m_c = 1.48 \text{ GeV}, \ |\mathcal{R}'(0)|^2 = 0.075 \text{ GeV}^5.$$

The pseudorapidity is defined as $\eta = \frac{1}{2}\ln\left((\sqrt{P_0^2 - M^2} + P_3)/(\sqrt{P_0^2 - M^2} - P_3)\right)$. To compare with data we multiply our cross sections with the braching ratio $Br(J/\Psi \to \mu^+\mu^-)$.

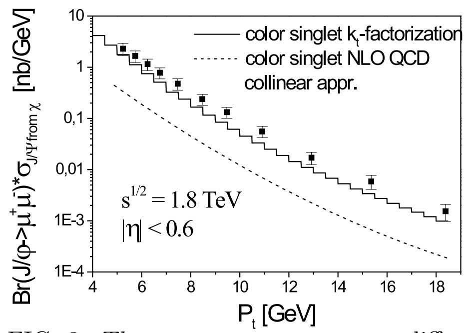

FIG. 3. The transverse momentum differential cross section in comparison to the data and a NLO QCD calculation

The resulting $P_\perp$-dependent cross section for J/Ψ's from radiative deacays of χ_c's produced in pp-collisions is shown in Fig.3 together with the data from the CDF Collaboration [6] and a NLO QCD collinear result (see Fig.7 in [11]). The individual contributions from χ_{c1} and χ_{c2} are shown in Fig.4.

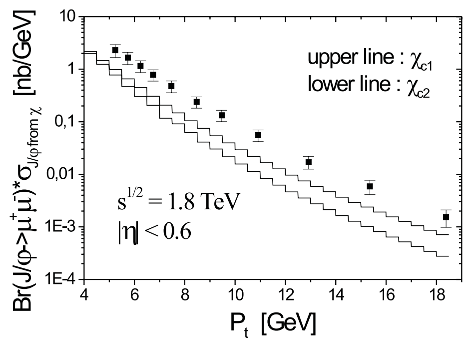

FIG. 4. The individual contributions from χ_{c1} and χ_{c2}

The description of the data is very satisfactory and becomes even better if the difference of the transverse momentum of J/Ψ (which is measured experimentally) and χ_c (which enters our calculation) is taken into account. (Due to the radiative decay the transverse momentum of J/Ψ is typically larger by an amount of ≈ 300 MeV than the corresponding χ_c one which leads to a shift of the theoretical curve to the right.)

We emphasize that the result has been obtained without fitting any of the parameters involved: The unintegrated gluon distribution has been adopted from Kwiecinski et al. [15]. The parameters of the quarkonium bound state are the ones given by Eichten and Quigg [16]. We found that the colour singlet production mechanism alone is sufficient to describe the Tevatron data, in particular the $P_\perp$ behaviour. For the χ_{c1} state it is crucial that the gluons are off-shell in the $k_\perp$ -factorization (inapplicability of the Landau-Yang theorem). The obvious conclusion is that the $k_\perp$-factorization approach with the BFKL production vertex provides the necessary improvement of the standard collinear approximation to QCD in the TeV range. There is actually no need for the colour octet production mechanism, which has been introduced just for the purpose to understand the Tevatron data in the collinear approach.

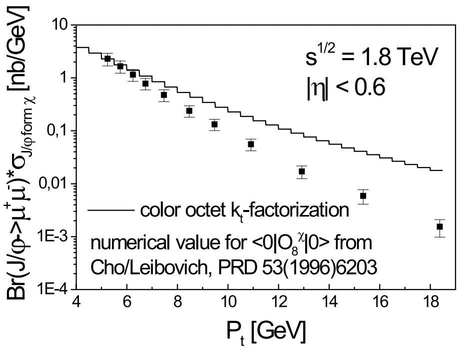

FIG. 5. The color octet contribution

The conclusion of the last paragraph is strongly supported by the following results of the analogous calculation of J/Ψ production from χ_c radiative decays adopting the colour octet mechanism instead. The χ_c state can be written in the velocity expansion as [10]

$$|\chi_c\rangle = \mathcal{O}(1)\,\left|q\overline{q}\left[{}^3P_J{}^1\right]\right\rangle + \mathcal{O}(\mathbf{v})\,\left|q\overline{q}\left[{}^3S_1^8\right]g\right\rangle + \cdots.$$

Following the formalism of [10,11] the resulting cross section is then proportional to the color octet matrix element $\langle 0|\, O_8^{\chi_{c1}}({}^3S_1)\,|0\rangle$ which has to be fitted to data. We calculate J/Ψ production from χ_c radiative decays with the latter being produced via the colour octet state ${}^3S_1^8$ and choose the value of the octet matrix element as in [11].

The resulting cross section from the color octet part only is shown in Fig.5. It is in striking disagreement with the data both in shape and normalization. We attribute this strong difference in the $P_\perp$ dependence of the octet and singlet contributions to the extra term in the NLLA vertex (the rightmost one in Fig. 2), which does not contribute to the production of singlet states, but shows up for the octet ones.

Let us conclude. The $k_\perp$-factorization approach relying on an unintegrated gluon distribution compatible with the small x behaviour of the structure function F_2 together with the BFKL NLLA fermion production vertices describes correctly χ_c production in the central rapidity region without any adjustable parameter. Whereas the standard collinear factorization approach in NLO can describe the data in the TeV range only by introducing the additional octet production mechanism involving soft gluon emission in the final state, we have shown that in the $k_\perp$-factorization approach such an additional contribution is even excluded by its $P_\perp$ behaviour.

Our main conclusion is therefore that the correct way to improve the standard QCD calculations for quarkonium production in the TeV range is to abandon the collinear approximation. The contributions disregarded in the collinear approximation of strong transverse momentum ordering become essential in the small-x range. The relative merits of the $k_\perp$ -factorization as the standard approach for other processes in high energy hadronic collisions still has to be investigated. But even if it would not be possible in heavy quarkonium production to abandon the color octet mechanism at all, we may expect that its contribution required to fit the data will be significantly suppressed in comparison to the collinear case.

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